Learning path for patent examiners

Mathematics and its applications:
Intermediate level

Version: July 2022
Introduction

This publication, "Mathematics and its applications, Intermediate level", is part of the "Learning path for patent examiners" series edited and published by the European Patent Academy. The series is intended for patent examiners at national patent offices who are taking part in training organised by the European Patent Office (EPO). It is also freely available to the public for independent learning.

Topics covered include novelty, inventive step, clarity, unity of invention, sufficiency of disclosure, amendments and search. Also addressed are patenting issues specific to certain technical fields:
- patentability exceptions and exclusions in biotechnology
- assessment of novelty, inventive step, clarity, sufficiency of disclosure and unity of invention for chemical inventions
- the patentability of computer-implemented inventions, business methods, game rules, mathematics and its applications, presentations of information, graphical user interfaces and programs for computers
- claim formulation for computer-implemented inventions

Each publication focuses on one topic at entry, intermediate or advanced level. The explanations and examples are based on the European Patent Convention, the Guidelines for Examination in the EPO and selected decisions of the EPO's boards of appeal. References are made to the Patent Cooperation Treaty and its Regulations whenever appropriate.

The series will be revised annually to ensure it remains up to date.

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# Contents

1. Learning objectives 4

2. What is a "mathematical method"? 4

3. First hurdle for mathematics 4

4. Mathematics: the two dimensions of the second hurdle 5

5. What is artificial intelligence (AI)? 7

6. AI: the two dimensions of the second hurdle 8

7. Simulation, modelling and design 9

8. Mathematics – examples 9

## Legal references

Art. 52(2) EPC; Art. 52 (3) EPC 4  
G-II, 3.3 5  
G-II, 3.3; G 1/19 6  
G-II, 3.3 6  
G-II, 3.3 7  
G-II, 3.3.1 8  
G-II, 3.3 9  
Article xxx 9  
G-II, 3.3.2 9  
G 1/19 9
1. **Learning objectives**

Participants to this course will learn:

- The definition of Mathematical Method and Artificial Intelligence
- To apply the two-hurdle approach for a MM and AI application
- The two dimensions in which features of a MM can contribute to the technical character of an invention
- The requirement of a specific technical purpose
- The requirement of a specific technical implementation and technical considerations regarding the internal functioning of the computer system

2. **What is a "mathematical method"?**

A mathematical method is a method that has steps that are mathematical computations or of an algorithmic nature, sometimes using formulae to describe what is computed.

While mathematical methods are non-inventions when taken "as such" (Article 52(a) and (3) EPC), they are at the core of technology and may well play a decisive part in achieving technical effects (G-II, 3.3).

The Fourier transform is an example of a mathematical method without which much technological progress would never have taken place. It is used in fields such as signal processing, optics and the analysis of protein structure.

**Examples**

Another example of a mathematical transform at the core of technological progress is the Radon transform, which enabled computer tomography once imaging devices became available.

**Legal references:**

Art. 52(2) EPC; Art. 52 (3) EPC

3. **First hurdle for mathematics**

The two-hurdle approach harmonises the interpretation of the exclusions by setting up two hurdles to be passed – the first, avoiding the exclusion from patentability, that is the claimed invention being eligible for patenting (Article 52(2) and (3) EPC), and the second, the claimed invention not being obvious when solving a technical problem (Article 56 EPC).

Given the key role of mathematics in technology in general, and that of AI/ML in particular, there are specific criteria for determining whether mathematical features in a claim contribute to technical character.

A distinction has to be drawn between claims in which mathematics are claimed "as such" versus claims in which a mix of technical and non-technical features is present: "mixed-type" inventions.

Section G-VII, 5.4 of the Guidelines sets out the problem-solution approach for claims comprising technical and non-technical features. All features contributing to the technical character are taken into account for assessing inventive step.
Section G-II, 3.3 specifies when a mathematical method (or mathematical method steps) can contribute to the technical character of the invention and thus needs to be taken into account when assessing inventive step in combination with the technical features.

A claim that is directed to a mathematical method can require technical means explicitly (typically "computer-implemented") or implicitly. Technical means may be implicit when steps of a method necessarily require the use of technical means. For instance, a step of "collecting/measuring an electromagnetic signal" requires technical means as it cannot be performed mentally.

When a claim is directed to a computer program, a further technical effect is required (G-II, 3.6) because running a program on a computer always has the – trivial – effect of moving electrons in the computer's circuits or accessing the computer's memory, for example. Normally, this further technical effect results from the corresponding computer-implemented method itself producing the further technical effect.

A claim that is "pure maths" with no technical means involved will be objected to as being excluded from patentability under Article 52(2) EPC. However, Article 52(2) EPC is a non-exhaustive list of non-inventions, and an abstract mathematical object (a polyhedron, for example) can also be considered a non-invention under Article 52(1) EPC.

If some sort of mathematics is performed on technical or physical parameters but again with no technical means involved, the claim will be objected to under Article 52(2)(c) and (3) EPC because it represents a mental act "as such" – the claim can be taken to be mere calculation instructions without using technical means.

Importantly, the fact that the parameters of a mathematical method have technical character does not imply the use of technical means.

Legal references:
G-II, 3.3

4. Mathematics: the two dimensions of the second hurdle

To avoid exclusion from patentability, the claimed subject-matter must have technical character when considered as a whole. This is sufficient to pass the first hurdle.

When assessing inventive step (Article 56 EPC), i.e. the requirements of the second hurdle, it must be ascertained which features of the claimed subject-matter contribute to technical character as only these may support the presence of an inventive step.

If the invention involves a mathematical method, the question arises as to whether the features of the claimed method defining this mathematical method contribute to the technical character of the invention.

There are essentially two orthogonal "dimensions" along which a mathematical method may make this kind of technical contribution (G-II, 3.3 and 3.3.1).

A mathematical method may contribute to the technical character of an invention, i.e. contribute to producing a technical effect that serves a technical purpose, by its application to a field of technology and/or by being adapted to a specific technical implementation.
For technical applications, when assessing the contribution made by a mathematical method to the technical character of an invention, it must be taken into account whether the method, in the context of the invention, serves a technical purpose.

For technical implementations, a mathematical method may also contribute to the technical character of the invention independently of any technical application when the claim is directed to a specific technical implementation of the mathematical method and the mathematical method is particularly adapted for that implementation in that its design is motivated by technical considerations regarding the internal functioning of the computer.

Provided that the mathematical method serves a technical purpose and/or is particularly adapted for a specific technical implementation, resulting technical effects such as improved processing speed can indicate the presence of an inventive step.

Legal references:
G-II, 3.3; G.1/19

Mathematics: the first dimension of the second hurdle

For the dimension of technical application, as mentioned above the relevant question is whether the mathematical method serves a technical purpose.

The most common situation is when the claim explicitly or implicitly specifies how the output is used. In that case, all that needs to be determined is whether this use is technical. However, there are other cases where the technical effect does not rely on the use of the output of the method.

A claim should be functionally limited to its purpose, whether explicitly or implicitly. Additional specifications as to how the input and output relate to the purpose are normally necessary to establish the contribution of the mathematical steps to technical character.

The purpose should be specific, i.e. not generic and pro forma, e.g. "controlling a technical system".

Specifying that the input to the mathematical method is measured data/physical data it does not necessarily imply that the mathematical method contributes to the technical character of the invention.

Legal references:
G-II, 3.3

Mathematics: the second dimension of the second hurdle

"A mathematical method for distributing load in a computer network" serves a technical purpose. The mathematical method is used for a concrete process in a technical system. The actual processing load of servers is dependent on the output of a mathematical method.

"A computer-implemented method of designing an optical system using a mathematical formula to determine refractive indices and magnification factors so as to obtain optimal optical performance" serves a technical purpose. The mathematical method results in refractive indices and magnification factors in a designed system having optimal optical performance under given input conditions.

No manufacturing step is required as that result is already technically relevant. Cases such as these are more "borderline".
"A cryptographic computation method with masking operations to protect the computation against power analysis" serves a technical purpose. Power analysis attacks involve measuring the power consumption during cryptographic operations. The encryption key may be found by performing a statistical analysis on the measurements. In this example, masking operations that do not change the result of the cryptographic computation are added, i.e. the output of the cryptographic method stays the same. Hence, the result of the masking operations is meant not to affect the output of the mathematical method, but to protect the cryptographic operation against a very concrete attack. A result such as this is of direct technical relevance.

However, a "method for classifying records comprising mathematical steps, the classified records being used in a billing procedure" serves a business purpose, not a technical purpose. Automatically classifying data records serves merely to classify the data records without implying any technical use of the classification. The claim covers any non-technical (e.g. administrative or commercial) use of the classified data records.

Legal references:
G-II. 3.3

5. What is artificial intelligence (AI)?

AI is a broad sub-field of computer science that covers many different computational models for solving data analysis problems. Our focus is on the machine learning branch of AI, but AI also includes other branches such as symbolic AI, which are present in what are known as "expert systems".

AI/ML algorithms/models are supported by advanced mathematics and are themselves of an abstract mathematical nature. This is why the topic of AI and machine learning is subsumed under mathematical methods in the Guidelines.

There are four main data analysis problems tackled by non-symbolic AI: classification, clustering, regression and dimensionality reduction.

- Classification is about identifying the category of a new observation on the basis of a labelled training set with categorised observations by which a classifier (a classification algorithm/model) has been trained. Examples of classification are identifying a file as being infected with a virus (according to certain characteristics), or determining whether to buy or sell stock in real-time depending on the current stock chart and a training set from top-trader past activities.
- Clustering aims at grouping sets of (data) objects such that objects in one cluster are more similar to each other than those in another cluster. An example of clustering is identifying gene families in a DNA sample.
- Regression is estimating a relationship from among variables of a dataset, typically by fitting a curve to the dataset.
- Dimensionality reduction attempts to reduce the number of variables characterising a dataset while retaining (at least some of) its information content.

The basic purpose of models tackling classification, clustering, regression, dimensionality reduction, etc. is abstract. The fact they are "trained" by a training algorithm does not change this (G-II, 3.3.1).
Caution is required in the field of AI/ML due to the use of field-specific terminology that may sound like it implies technical means ("machine" in support vector machine or "network" in neural network) but in fact generally refers to abstract models devoid of technical character (G-II, 3.3.1).

The take-home message here is that the principles of examination regarding mathematical methods also apply to AI and ML.

Legal references:
G-II, 3.3.1

6. AI: the two dimensions of the second hurdle

Like with mathematical methods in general, the issue for methods involving AI is also whether the AI and ML method (or method steps) contributes to the technical character of the invention.

The AI and ML method (or method steps) contributes to the technical character of the invention to the extent that, in the context of the invention, a technical purpose is served by the technical application of the (overall) method, i.e. to solve a technical problem in a technical field.

Prominent examples of technical applications of AI/ML are found in the fields of speech/image processing, fault detection and engine control, for example.

Classification for determining the price of a service is a typical business application, not a technical application.

The requirements of G-II, 3.3 apply.

Example of the second dimension of the second hurdle

An enhanced classifier for classifying digital images on the basis of an expanded training set

FIG. 9
This example is based on case T.1286/09 in the field of image classification.

The purpose in this case was to classify pictures as "sunset", "beach", "forest", "picnic" or the like. In the prior art, doing so was found to be difficult because there were a huge variety of "sunset" images.

In this invention, an improved classifier was implemented by adapting the training data. Given a set of images for training, the set was first expanded by varying the colour codes slightly and gradually, to produce more training data.

In the end, the training dataset was expanded, and the resulting better trained classifier was found to perform much better and to be much more reliable.

Legal references:
G-II, 3.3

7. Simulation, modelling and design

"This section will be revised in view of G.1/19 and the new Guidelines that will come into force on 1st of March 2022".

Legal references:
Article xxx
G-II, 3.3.2
G.1/19

8. Mathematics – examples

Example 1

An example with the Fast Fourier Transform and typical concrete claim wording comprising mathematical features is provided below. There is no need to go into the mathematical detail.

A method for calculating a Fast Fourier Transform $y$ of vector $x$, wherein $y = F_n \ast x$, where $F_n = \left( \begin{array}{ccc} \vdots & \cdots & \vdots \\ \omega^0 & \cdots & \omega^{n-1} \end{array} \right)$ and $y$ is calculated by calculating $u = F_{n/2} \ast \left( \begin{array}{c} x_0 \\ x_2 \\ \vdots \end{array} \right)$ and $v = F_{n/2} \ast \left( \begin{array}{c} x_1 \\ x_3 \\ \vdots \end{array} \right)$ and calculating each $y_i$ using $y_i = \left\{ \begin{array}{ll} u_i + \omega_n^i v_i & \text{for } 0 \leq i < n/2 \\ u_{i-n/2} + \omega_n^i v_{i-n/2} & \text{for } n/2 \leq i < n \end{array} \right.$

It should first be noted that no computer-implementation is claimed, so no technical means are involved.
The method manipulates purely abstract input parameters, i.e. the vector \( x \), the matrices \( F_n \), \( F_{n/2} \) and the resulting vector \( y \).

The method thus remains purely abstract and devoid of technical character. An objection should thus be raised under Article 52(2)(a) and (3) EPC.

**Example 2**

In the slightly modified version of the previous claim below, it is added that \( x \) represents a series of temperature measurements. Therefore, \( x \) is no longer an abstract parameter. In other words, the method is already mathematics "applied" to concrete physical parameters.

```plaintext
A method for calculating a Fast Fourier Transform \( y \) of vector \( x \), comprising

\[ ... [formulas] ... \]

wherein \( x \) represents a series of temperature measurements.
```

```plaintext
A method for calculating a Fast Fourier Transform \( y \) of vector \( x \), comprising

\[ I \]

\[ ... [formulae] ... \]

wherein \( x \) represents a series of temperature measurements.
```

Since no technical means are required and it is thus still possible to construe the claim as an instruction to the human mind on how to calculate a transform, an objection under Article 52(2)(c) and (3) EPC is raised (the claimed method encompasses a mental act as such). This illustrates that specifying only the input parameters of a mathematical method does not necessarily overcome the first hurdle.

**Example 3**

With additional restrictions compared with the previous claim (see below), with the measurements now being obtained using a sensor, the claim requires technical means and is no longer excluded from patentability.

```plaintext
A method for calculating a Fast Fourier Transform \( y \) of vector \( x \), comprising

- obtaining a series of measurements **using a temperature sensor**
- assigning the obtained series of measurements to \( x \)
- calculating \( y \) using

\[ ... [formulas] ... \]
```
Example 4

The invention concerns a "polynomial reduction operation" (T.1925/11). For a modulus of high degree (multi-word), the operation can be performed with word shifts rather than bit shifts. To this end, the formulae used are reformulated in terms of the "word size w", more precisely in terms of divisions by $x^{2k+w}$ and $x^{k-w}$.

Without going into the complex mathematical details of this example, the key message here is that the mathematical operations performed are specifically adapted to the underlying architecture that offers word shift operations.

From point 8 of the Reasons in the aforementioned case, it is clear that the board considers that the implementation of the algorithm in terms of word shifts (of the underlying hardware) contributes to inventive step, and thus implicitly to technical character.

This is said to "simplify handling of the polynomial quantities on computational hardware".